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THE OPTIMIZATION PROBLEM OF THE RETIREMENT RATE IN THE CONTEXT OF THE TRANSITION TO NON-STATE PENSION SYSTEM

In recent decades, a market economic system has been forming at the arena of the post-socialist countries. One of the most important issues of modern social and economic relations is the issue of pensions accruals and payment. In this context,it's urgent to develop the measures to reform the PAYG system, which isn't valid anymore in Ukraine. The first problem is the need to calculate the amount of pension contributions that would provide a more effective functioning of the system.

Non-pension system allows one to create large amounts of financial resources, and place them, while providing high pensions paid to the fund members.

The main problem of post-socialist countries in this sphere is not the full absence of such institutions, but moreover their inefficiency. This is connected, firstly, to the distrust of most citizens to organizations providing any financial services, and secondly –to the pension contributions, which are too high comparing to the amount of income and therefore such services are inaccessible to the majority of the population.

The issue of distrust may not be solved as quickly and easily as we would like, but one of the options to resolve it a complete transition to a non-state pension system would be. In such anon-state pension system there would be assets, financial resources, sufficient enough to ensure that the participants don't doubt their reliability. The other issue is more complicated, such as improving the competitiveness of private pension funds. This problem is very complex and requires applying a systematic approach to solve it, but we would like to emphasize the importance of the pension contributions, as they are one of the main criteria of the pensions choice.

Let us consider the hypothetical pension fund in which N participants aged x years bring some amount of money – z. At the time of retirement the amount the participant gets is equal to:

$$S = z(1+i)^t.$$

Taking into consideration that not all pool participants reach the retirement age x + t, the participants who survive can increase their revenues by people who are not of the appropriate age. The number of people who will live to the date of payment can be found as follows: $N \times P(x < x+t) = \frac{l_{x+t}}{l_x}$, where $P(x < x+t)$ – distribution function of a random variable; probability that a person aged *x* years will live until the *x* + *t*, P$\in(0,1)$; l_x, l_{x+t} – the number of participants at the beginning of the period and at the time of payment.

Thus, due to participants who will not have lived up to the time of payment, the rest of pensioners will receive the same amount:

$$S^* = \frac{l_x}{l_{x+t}} z(1+i)^t$$

To determine the level of contributions required of the discount given by the sum of i:

$$X = \frac{S^*}{(1+i)^t} \times \frac{l_{x+t}}{l_x}$$

The fact that $X \ll z$ is quite obvious .Thus due to the participants who will not have lived up to the time of payment we can reduce the size of the pension contribution. But this is not the way out, as the pension fund can not and should not affect the mortality in order to get the pensions increase. However, there is one parameter model which could theoretically affect the pension fund – it's the discount rate. On the one hand i- is the opportunity cost of fundraising in determining the level of contributions, on the other – a rate of return in determining the payments at time *x* + *t*.

Since the interest rate is the rate of return and then it would be great to find a way to zoom to a certain maximum limits. Accumulating Pension Fund contributions creates an investment portfolio so as to ensure a certain level of benefits. Today we know a lot of models for optimizing the portfolio. The very first and the most well-known model was the one of Harry Markowitz.

The method is as follows:

- each investor has a utility function $\sigma, \overline{r_i}$), which depends on the portfolio risk and return and σ– On a financial asset (r_i)̅;
- investor chooses for himself a combination of securities held;
- set of portfolios that investors can choose a strictly convex; $\frac{dU}{d\sigma} < 0, \frac{dU}{d\overline{r_i}} > 0$, i. e. the investor's marginal utility of risk decreases, and the yield increases;
- full differential utility function of the investor is:

$$dU = \frac{\partial U}{\partial \overline{r_i}} d\overline{r_i} - \frac{\partial U}{\partial \sigma} d\sigma = 0 \leftrightarrow \frac{d\overline{r_i}}{d\sigma} = \frac{\frac{\partial U}{\partial \sigma}}{\frac{\partial U}{\partial \overline{r_i}}}$$

since $\frac{d\bar{r_i}}{d\sigma} > 0$, i. e, the marginal rate of substitution risk of yield increases, the map of indifference curves investor has a positive slope, in fact they are also strictly convex;

- the investor chooses a portfolio that meets the following conditions:

$$Q^* = W\{q|r > r_{\min}\} \cap F\{q|\sigma < \sigma_{\max}\},$$

set Q^* is not empty, in fact, it consists of the portfolios that contain the maximum yield and minimum risk.

Therefore the analytical problem narrows to maximizing the investor's objective function yield securities portfolio under the given constraints, so the problem of the investor is:

$$R_p = \sum_{i=1}^{n} w_i \times \bar{r_i} \to \max$$

$$\sum_{i=1}^{n} w_i = 1,$$

$$D_p = \sum_{i=1}^{n} w_i^2 \times D_i + 2\sum_{i \neq j} w_i \times w_j \times cov_{ij} = D_p^*$$

where w_i – the proportion of funds invested into the land and financial assets; R_p – yield securities portfolio; D_p – variance portfolio.

But before using this method for solving the optimization problem of the pension rate, one should determine the conditions under which R_p and both are equivalent. Equivalence means that the interest rate should be either similar or identical or vary in the same direction and so on. Since the interest rate is a complex function that depends on many variables, and we can not objectively consider them, and do not need it in this in these conditions, so we can summarize all the factors that affect these functions using a single variable –the time. We may write the necessary and sufficient conditions for the equivalence of interest rates as follows:

1. $\left[\frac{dR_p}{dt} > 0 \cap \frac{di}{dt} > 0\right] \cup \left[\frac{dR_p}{dt} < 0 \cap \frac{di}{dt} < 0\right]$
2. $\frac{dR_p}{dt} = \frac{di}{dt}$.

Now let’s go directly to solving the investor’sproblem. It is given that the objective function of return on the portfolio given a limited and closed sets, there is an extremum of this function and it is only because of strict vypuklosti utility function of the investor and the set of efficient portfolios.Let us write the Lagrangian function for the pension fund:

$$L(w_1, \dots, w_n, \lambda_1, \lambda_2) = w_1 \times \bar{r_1} + \cdots + w_n \times \bar{r_n} + \lambda_1 (w_1 + \cdots + w_n - 1) + \lambda_2 (w_1^2 \times D_1 + \cdots + w_n^2 \times D_n + 2(cov_{12} \times w_1 \times w_2 + \cdots + cov_{1n} \times w_1 \times w_n + cov_{2n} \times w_2 \times w_n + \cdots + cov_{(n,n-1)} \times w_n \times w_{(n-1)}) - D_p^*) \to max$$

The necessary condition for finding an extremum of this function is vanishing of all its partial derivatives, this way:

$$\frac{\partial L}{\partial w_i} = 0, \forall i \in \{1, \dots, n\}$$

$$\frac{\partial L}{\partial \lambda_k} = 0, \forall k \in \{1,2\}$$

So we got a system with $n + 2$ linear equations that can be solved applying the matrix method:

$$w = A^{-1} \times b,$$

where w – the column vector of unknown variable; A^{-1} – the matrix inverse to the matrix of coefficients of the variables; b – column absolute terms.

So we found the proportion that should occur in each financial asset in the portfolio securities in order to get the maximum yield for a given level of risk. The question still remains for which the maximum is willing to take the risk of the pension fund for the reduction in the size of the regular pension contributions.

The solution to this problem, increase the competitiveness of private pension funds will help get rid of the problem of low pensions and the burden on the state budget. In addition, the system of financing pension benefits gives the economy of low-cost funding for the implementation of investment, and can improve the level of consumption in the national economy. So the use of the abovementioned scheme may be a solution to many problems and would be beneficial for all parties of such relationships.